Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Back Exam – 2081 Phagun/Chaitra

Program:	**Diploma in Civil/Information Technology/ Computer/Agricultural/Engineering**	**Full Marks: 80**
Year/Part:	**II/I (2021, 2022)**	**Pass Marks: 32**
Subject:	**Engineering Mathematics III**	**Time: 3 hrs.**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt ALL questions. **[(7×2)×2=28]**

1. a. Find f_x, f_y and f_z where $f(x,y,z) = xy + yz + zx$.

   b. Prove that the function $f(x,y) = x^3 + 3x^2y - y^3$ is homogeneous. Also, find its degree.

2. a. Find $\frac{dy}{dx}$ when $y = \log(\tanh x)$.

   b. Find the derivative of $y = \cos^{-1}(3x - 4)$.

3. a. Using L-hospital rule to evaluate: $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$.

   b. Find the equation of the tangent to the curve: $y = 2x^3 - 5x^2 + 8$ at (2, 4)

4. a. Evaluate: $\int \frac{dx}{4x^2+25}$

   b. Find the area bounded by the curve $y = x^2$ with the x-axis and two ordinates x=1, x=3.

5. a. Integrate: $\int \sqrt{25 - 9x^2}\, dx$

   b. Find the fundamental period of tan2x.

6. a. Solve: $\tan x\, dy + \tan y\, dx = 0$

   b. Form a PDE eliminating a and b from $z = ax + by + ab$.

7. a. Solve: $p + q = 1$

   b. Solve: $\frac{dy}{dx} + 1 = e^{x+y}$

Cont. ……

Group 'B'

Attempt ALL questions. **[13×4=52]**

8. Find the local maxima, local minima and point of inflection of the function $f(x) = 2x^3 - 15x^2 + 36x + 5$.

OR

Find the two numbers whose sum is 10 and the sum of whose square is minimum.

9. Water is poured into a right circular cylinder of radius 6 ft. at the rate of 15 cu. ft./min. Find the rate at which the level of water is rising in the cylinder.

10. Show that the equation of the tangent to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$

11. Use definition to find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x, y) = xy + y^2$

12. Find $\frac{du}{dt}$ when $u = x^2 + y^2, x = \cos t + \sin t, y = \cos t - \sin t$ at t=0.

13. Find the area of circle $x^2 + y^2 = 9$.

OR

Sketch the graph of curve $y = x^2 - 4x + 3$

14. Find the area of the region between the curves $y^2 = 16x$ and line $y = 2x$.

15. Integrate: $\int \frac{dx}{3\sin x + 4\cos x}$

16. Solve: $(x + y + 1)\frac{dy}{dx} = 1$

17. Show that the equation: $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$ is exact and solve it.

18. Form a PDE by eliminating the *f* from: $\ell x + my + nz = f(x^2 + y^2 + z^2)$

19. Solve: $xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = xy$

20. Find the Fourier series of the function:

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \leq 1 < \pi \end{cases}$$

Good Luck !

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Back/Scholarship Exam – 2081/2082 Chaitra/Baishakh

Program:	**Diploma in Civil/Electronics/Architecture/ Information Technology/Hydropower/ Computer Engineering**	**Full Marks: 80**
Year/Part:	**II/I (2013, 2014, 2016, 2017, 2018)**	**Pass Marks: 32**
Subject:	**Engineering Mathematics III**	**Time: 3 hrs.**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt ALL questions. **[(5×2)×3=30]**

1. a. Define periodic function and fundamental period with example. Find the fundamental period of tan5x. [1+1+3]

   b. Find the Fourier series of the function: [5]

$$f(x) = \begin{cases} 0 & \text{for } -\pi < x < 0 \\ 1 & \text{for } \ 0 < x < \pi \end{cases}$$

2. a. Define abelian group. Prove that inverse of any element is a group is unique. [5]

   b. Prove that the fourth roots of unity, $S = \{1, -1, i, -i\}$ forms a group under multiplication. [5]

3. a. Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x, y) = x^2y - xy^2$. [5]

   b. Solve the homogeneous differential equation: [5]

$$\frac{dy}{dx} = \frac{x^2+y^2}{2x^2}$$

Group 'B'

Attempt ALL questions. **[10×5=50]**

4. Solve: $\frac{dy}{dx} = \frac{x+y}{x}$

5. Solve the partial differential equation:

$$z = ax + by + a^2 + b^2$$

6. Solve: $\sin x \cos x \, dx + \sin y . \cos y \, dy = 0$

7. Test the convergence of the series by using Cauchy's Root Test: $1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \cdots$ for all positive value of x.

Cont. ……

8. Show that series: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is conditionally convergent.

9. Find the interval and radius of convergence of power series: $1+2x+4x^2+8x^3+\ldots\ldots$

10. Find the Maclaurin's series expansion of $cos x$.

11. Expand f(x) in the Fourier series if $f(x)=x^2$ $(0<x<2\pi)$.

12. If $u = \log \frac{x^3+y^3}{x+y}$ prove that $\frac{x\,\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$.

13. Prepare a Cayley's table for the set $\{0, 1, 2, 3, 4\}$ under the operation multiplication modulus Identify the identity and inverse element if exists.

Good Luck !

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Scholarship Exam – 2081 Mangsir

Program:	**Diploma in Civil Engineering**	**Full Marks: 80**
Year/Part:	**II/I (2021)**	**Pass Marks: 32**
Subject:	**Engineering Mathematics III**	**Time: 3 hrs.**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt ALL questions. [(7×(2×2)=28]

1. a. Find $\frac{dy}{dx}$ when $y = \sin^{-1}(3x^2 - 2)$.

   b. Find the derivative of $\ln\left(cosh\frac{x}{a}\right)$

2. a. Find the points on the curve $x^2 + y^2 = 16$ where tangent is parallel to x-axis.

   b. Evaluate: $\lim\limits_{x\to2}\frac{x^3-2x^2+2x-4}{x^2-5x+6}$

3. a. If $U = x^2 + y^2 + z^2$, show that $XUx + YUy + ZUz = 2U$.

   b. If $z = 3x^3y^3 - 9x^2y + xy^2 + 4y$, find partial derivatives of $\frac{\partial z}{\partial x}$ and their values at (1, 0).

4. a. Integrate: $\int\frac{dx}{\sqrt{x^2-2ax}}$

   b. Examine whether the function $f(x) = e^x - e^{-x}$ is even or odd.

5. a. Integrate: $\int\frac{dx}{4x^2-25}$

   b. Solve: $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$

6. a. Solve: $xdy + (x + y)dx = 0$

   b. Test the exactness of $(2ax + by)ydx + (ax + 2by)xdy = 0$.

7. a. Solve the differential equation of $\frac{dy}{dx} + \frac{y}{x} = 2x$

   b. Find the fundamental period of $f(x) = sin2x$.

Cont. ……

Group 'B'

Attempt ALL questions. [13×4=52]

8. Find the maximum and minimum values of the function $f(x) = x^4 - 14x^2 - 24x + 1$

9. Find the equation of tangent and normal to the curve $y = x^3 - 2x^2 + 4$ at the point (2,4).

10. Water flows into an inverted conical tank at the rate of 24 ft^3/min. when the depth of water is 9ft. How fast is the level rising? Assume that the height of the tank is 15 ft. and the radius at the top is 5 ft.

11. Use definition find $\frac{\partial f}{\partial x}$ *and* $\frac{\partial f}{\partial y}$ of $f(x, y) = x^2 - 3xy$.

12. State the Euler's theorem of homogeneous function. If $z = x^n \phi\left(\frac{y}{x}\right)$ then prove that: $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$

13. Find the area of the region between the curve $y^2=4x$ and $x=y$.

14. Evaluate: $\int \frac{dx}{sinx+cosx}$

15. Find the area of ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

16. Solve: $\frac{dy}{dx} = \frac{y}{x} + tan\left(\frac{y}{x}\right)$

17. Show that the differential equation exact and solved: $\frac{dy}{dx} = \frac{x-y+1}{x+y+1}$

18. If the normal at every point of a curve passes through a fixed point, show that the curve is a circle.

19. Form a Partial Differential Equation (PDE) by eliminating the function (F) from $x + y + z = F(x^2 + y^2 + z^2)$

20. Find the Fourier series of the functions:

$$f(x)=\begin{cases} -2x & \text{for } -\pi<x<0 \\ 2x & \text{for } 0<x<\pi \end{cases}$$

Good Luck !

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Back/Scholarship Exam-2080/2081, Chaitra/Baishakh

Program: Diploma in Civil/Hydropower/Information Technology/Computer/Agriculture Engg. **Full Marks: 80**

Year/Part: II/I (2021, 2022) **Pass Marks: 32**

Subject: Engineering Mathematics III **Time: 3 hrs.**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt ALL questions. [(2+2)×7=28]

1. a. Find the derivative of $\cos^{-1} x^2$

   b. The side of a square sheet is increasing at the rate of 5cm/min. At what rate is the area increasing when the side is 12cm long?

2. a. Show that the function $f(x) = 3x^3 - 24x + 1$ is increasing at $x = 4$ and decreasing at $x = \frac{1}{2}$

   b. Evaluate using L Hospital's rule:

$$\lim_{x\to 2} \frac{x^3 - 2x^2 + 2x - 4}{x^2 - 5x + 6}$$

3. a. Find first order partial derivatives of

$$f(x, y) = ln(2x + 5y)$$

   b. Verify Euler's theorem for the function; $f(x, y) = x + y$

4. a. Examine whether the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is even or odd?

   b. Evaluate: $\int \frac{1}{x^2+4}\, dx$

5. a. Find the area bounded by the curve $y = 2x^2$ x-axis and the ordinates $x = 0,\ x = 2$

   b. Write the engineering application of FEM.

6. a. Solve by separation of variable method of $\frac{dy}{dx} = 1 + y^2$

   b. Solve: $\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x}$

7. a. Solve: $x\frac{dy}{dx} + y = x^4$

cont. ....

b. Determine the order and degree of the differential equation:

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$$

Group 'B'

Attempt ALL questions. [13×4=52]

8. Find the derivatives of x^{coshx}

9. Find the local maxima, minima and point of inflection if exists;

$$f(x)=2x^3-3x^2-12x+4$$

10. Evaluate:

$$\lim_{x\to 0}\frac{tanx-x}{x^2.tanx}$$

11. Let $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that; $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=tanu$

12. If $u\,(x,y,z)=x^2+y^2+y^2$, $x=2t+1, y=t+5$ *and* $z=7t$ then find $\frac{du}{dt}$

13. Evaluate: $\int(x-3)\sqrt{x^2-1}\,dx$

14. Using integration, find the area of circle $x^2+y^2=a^2$

15. Using limit of the sum, evaluate: $\int_0^1 x^2dx$

16. Define linear differential equation and solve:

$$\frac{dy}{dx}+2ytanx=sinx$$

17. Show that the differential equation is exact and solve it.

$$(x+y-1)dx+(x-y-2)dy=0$$

18. The population growth rate of a certain town is 8% per year. Model the situation using a differential equation. What will be the population after 10 years?

19. Solve: $p+q=x$

20. Use the finite difference method to solve
$y''=y+x(x-4)\qquad 0\le x\le 4$
with $y(0)=y(4)=0$

Good Luck !

Council for Technical Education and Vocational Training
Office of the Controller of Examinations
Sanothimi, Bhaktapur
Back/Scholarship Exam-2080/2081, Chaitra/Baishakh

Program: Diploma in Civil/Hydropower/Architecture /Information Technology/Computer/ Electronics Engineering **Full Marks: 80**

Year/Part: II/I (2013, 2014, 2016, 2017, 2018) **Pass Marks: 32**

Subject: Engineering Mathematics III **Time: 3 hrs.**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt ALL questions. [(5×2)×3=30]

1. a. Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y) = ax^2+2hxy+by^2$.

   b. Define homogenous function. State and prove Euless theorem for a two variable cases.

2. a. Solve: $\frac{dy}{dx} = 1 + \tan(y - x)$

   b. If $u = sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = tanu$.

3. a. Define Fourier series of a function f(x) on the interval $(-\pi, \pi)$. Find the Fourier series of:

$$f(x)=\begin{cases} -1 & -\pi<x<0 \\ -1 & 0\leq x<\pi \end{cases}$$

   b. Find the MacLaurin's series expansion of sinx.

Group 'B'

Attempt ALL questions. [10×5=50]

4. Solve: $(xy^2+x)dx+(yx^2+y)dy=0$

Cont. ......

5. Form a partial differential equation: $lx+my+nz=f(x^2+y^2+z^2)$

6. Solve the partial differential equation:

$$x(y-z)\frac{\partial z}{\partial x}+y(z-x)\frac{\partial z}{\partial y}=z(x-y)$$

7. Find $\frac{du}{dt}$ where, $u = z - \sin xy, x = t, y = \log t, z = e^{t-1}$ at t=1.

8. Test the convergence of series

$$\sum_{n=1}^{\infty}\frac{3^n}{n!}$$

9. Discuss the convergences of the series:

$$x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+\cdots+\frac{n^2-1}{n^2+1}x^n+\cdots$$

10. Find the interval of convergence and the radius of convergence of the power series.

$$\sum_{n=0}^{\infty}\frac{(x-2)^n}{10^n}$$

11. Define periodic function. Find the fundamental period of $\cos 2\pi x$.

12. Define a group. In a group prove that $(a*b)^{-1}=b^{-1}*a^{-1}$. Also, prove that inverse of each element of a group is unique.

13. Define binary operation. Show that multiplication (x) is binary on the set $S=\{1, w, w^2\}$ where w is cube root of unity.

Good Luck !

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Scholarship Exam-2080 Bhadra

Program:	**Diploma Civil/Hydropower/Information Technology/Computer Engineering**	**Full Marks: 80**
Year/Part:	**II/I (2021, 2022)**	**Pass Marks: 32**
Subject:	**Engineering Mathematics III**	**Time: 3 hrs.**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group'A'

Attempt All questions. **[(7×2)×2=28]**

1. a) Find the derivative of $\log(\tan 2x)$.

   b) Find the derivative of $x^{sinh\frac{x}{a}}$.

2. a) Using L – Hospital rule: Evaluate: $\lim_{x\to 0}\frac{logtanx}{logx}$

   b) Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangent are parallel to $x - axis$.

3. a) If $f(x,y) = \sqrt{x^2 + y^2}$, then find fxx at the point (2, 1).

   b) Find $\frac{du}{dx}$ if $u = x^2 + y^2, x = at^2$ and $y = 2at$.

4. a) Evaluate: $\int \frac{dx}{e^x+e^{-x}}$

   b) Examine whether the function $f(x) = \frac{e^x-e^{-x}}{e^x+e^{-x}}$ is even or odd.

5. a) Evaluate: $\int \sqrt{2ax - x^2}\, dx$

   b) Solve: $\sqrt{1-x^2}\, dy + \sqrt{1-y^2}\, dx = 0$

6. a) Solve: $x\, dy + y\, dx = 0$

   b) Form the partial differential equations: $z = ax + by + a^2 + b^2$

7. a) Solve: $\frac{dy}{dx} = e^{x-y} + x^3e^{-y}$

   b) Find the fundamental period of $f(x) = sin2\pi x$.

Group 'B'

Attempt ALL questions. **[13×4=52]**

8. Find the maximum and minimum values of the function $f(x)$=x^3- 6x^2+ 9x-2.

Cont. ....

OR

A man wishes to fence a rectangular garden with 256-meter fencing material. Find the maximum area he can enclose.

9. A spherical ball of salt dissolving in water decreases its volume at the rate of 0.75 cm^3/min. Find the rate at which the radius of the salt is decreasing when its radius is 6 cm.

10. Find the equation of the tangent and normal to the curve $y = x^3 - 2x^2 + 4$ at (2, 4).

11. Use definition. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x, y) = x^2y - xy^2$.

12. Verify Euler's theorem for homogeneous function $f(x, y, z) = x^2 + y^2 + z^2$.

13. Sketch the graph of $y = (x - 1)(x - 2)$.

OR

Find the area of circle $x^2 + y^2 = 36$.

14. Evaluate: $\int \frac{dx}{1-2cosx}$

15. Using limit of the sum, find the area bounded by the curve $y = 3x^2$, the x-axis and the ordinates $x = 0$ and $x = 4$.

16. Solve: $\frac{dy}{dx} = \frac{y}{x} + tan\left(\frac{y}{x}\right)$.

17. Solve: $(1 + x^2)\frac{dy}{dx} + 2xy = 3x^2$.

18. The half-life of isotopic radium is 300 years. Find the time required to decay 10% of its initial amount.

19. Solve: $xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = xy$.

OR

Form the partial differential equations if $x + y + z = f(x^2 + y^2 + z^2)$.

20. Find the Fourier series of the function:

$$f(x) = \begin{cases} 1 & -\pi < x < 0 \\ -1 & 0 \le x < \pi \end{cases}$$

Good Luck !

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Back/Scholarship Exam-2080 Bhadra

Program: **Diploma Civil/Architecture/Electronics/ Information Technology/ Hydropower /Computer Engineering** **Full Marks: 80**

Year/Part: **II/I (2013, 2014, 2016, 2017, 2018)** **Pass Marks: 32**

Subject: **Engineering Mathematics III** **Time: 3 hrs.**

Candidates are required to give their ans
figures in the margin indicate full marks.

Group'A'

Attempt All questions. **[(5×2)×3=30]**

1. a. Define Fourier series of a function f(x) on the interval $(-\pi,\pi)$. Find the Fourier series of:

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \le x < \pi \end{cases}$$

b. Test whether the function $f(x)=x^2, -1<x<1$ is even or odd. Also, find the appropriate Fourier series.

2. a. Define group. The identity element in a group is unique. Prove.

OR

A set of matrices of the form $A_\theta = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$ where θ is a number, is given:

i. Show that the operation of matrix multiplication is closed.

ii. Show that A_0 is the identity element of A_θ.

iii. Show that $A-\theta$ is the inverse element of A_θ.

b. Let G = {0, 1, 2}, form a composition table for G under multiplication modulo 3. Find the identify and inverse element of 1 and 2.

3. a. Using definition method, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y)=x^2y^2$.

b. Let $u = sin^{-1}\frac{x^2+y^2}{x+y}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

Group'B'

Attempt any TEN questions. **[10×5=50]**

4. By separating variables, solve: $(xy + x)dy - (xy + x)dx = 0$

Cont. ……

5. Change the equation $x^2y\ dx - (x^3 + y^3)dy = 0$ into homogeneous differential equation and solve it.

6. Show that the equation is exact and solve it:
   $(x + y - 1)dx + (x - y - 2)dy = 0$

7. Form a partial differential equation by eliminating 'f' from:
   $lx + my + nz = f(x^2 + y^2 + z^2)$

8. Solve : $\frac{dy}{dx} = \frac{y}{x} + tan\ \frac{y}{x}$

9. By using D' Alembert's ratio test, test the convergence or divergence of the series: $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots$

10. Test whether the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is absolutely convergent of conditionally convergent.

11. Using Maclurin's series, expand the function $f(x) = Sinx$

12. Find the interval of convergence and radius of convergence of the power series:

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n(n+1)}{2}x^{n-1}$$

13. Define periodic function. Find the fundamental period (P) of $f(x) = Sin2x$.

14. Test the following series for convergence by Cauchy's root test:
   $\frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)x^3 + \cdots + \left(\frac{n+1}{n+2}\right)x^n + \cdots$ for x ≠ 1.

Good Luck !

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Back Exam-2079, Bhadra/Ashwin

Program:	**Diploma Civil /Computer/Electronics /Architecture/IT/ Hydropower/Engg.**	**Full Marks: 80**
Year/Part:	**II/I (2013, 2018, 2014, 2016, 2017)**	**Pass Marks: 32**
Subject:	**Engineering Mathematics III**	**Time: 3 hrs.**

Candidates are required to give their ans... figures in the margin indicate full marks.

Group'A'

Attempt All questions. **[3×(5+5)=30]**

1. a) Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ when

$$f(x,y) = x^3 + y^3 + 3axy$$

b) Find $\frac{df}{dt}$ of $u = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$.

2. a) Define Group. Prove that the identity element of group is unique. Also show that the inverse of group is unique.

b) If $G = \{\cdots\cdots 6, -4, -2, 0, 2, 4, 6 \cdots\cdots\}$ then prove that (G, +) is a group.

3. a) Test whether the following series is absolutely or conditionally convergent:

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots\cdots$$

b) Find the Taylor's series expansion of $f(x) = e^{-x}$ about $x = 2$.

Group'B'

Attempt All questions. **[10×5=50]**

4. Solve by separating the variables :

a) $e^{x-y}dx + e^{y-x}dy = 0$ b) $\frac{dy}{dx} = -\frac{1+\cos 2y}{1-\cos 2x}$

5. Solve the homogeneous differential equation : $\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$.

Cont………

6. Find the Fourier series expansion of $f(x) = \begin{cases} 0 & -\pi < x < \\ 1 & 0 \leq x < \pi \end{cases}$.

7. Define periodic function. Find the smallest positive period of P of sinnx.

8. Prepare Cayley table for the set {0,1,2,3,4,5} under the operation Multiplication module 6. Identify the identity element and the inverse of each element if possible.

9. Solve the partial differential equation : (Any One)

i) $\frac{\partial f}{\partial x} xz + yz \frac{\partial f}{\partial y} = xy$. ii) $x p - yq + x^2 - y^2 = 0$

10. Find the interval and radius of convergence of the series :

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n^2}$$

11. Verify Euler's theorem for homogeneous function

$f(x, y, z) = x^2 + y^2 + z^2$.

12. Define convergent and divergent series. Determine whether the following series is convergent of divergent by ratio test

$$\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots\cdots$$

13. Test whether the function is even or odd. Find the corresponding Fourier series

$$f(x) = \begin{cases} \pi, & -1 < x < 0 \\ -\pi, & 0 \leq x < 1 \end{cases}$$

Good Luck!

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Back Exam-2078, Kartik/Mangsir

Program: Diploma in Civil/Hydropower/Architecture/ Electronics/IT/Computer Engineering **Full Marks: 80**

Year/Part: II/I (2013, 2017, 2014, 2016, 2018) **Pass Marks: 32**

Subject: Engineering Mathematics - III **Time: 3 hrs**

Candidates are required to give their answers in their own words as far as practicable. The figures in the m

Group 'A'

Attempt All questions. **[(5+5)x3=30]**

1. a) Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x,y) = x^2y - xy^2$

   b) $If\ u(x,y,z) = x^2 + y^2 + z^2, x = 2t + 1, y = t + 5\ and\ Z = 7t,$ $then\ find \frac{du}{dt}$

2. a) State limit comparisons test and use it to test the convergent or divergent of the infinite series.

   $\sum \sqrt{n^2 + 1} - n$

   b) Find the Fourier series of the function

   $$f(x) = \begin{cases} 1 & -\pi < x\ 0 \\ -1 & 0 \leq x < \pi \end{cases}$$

3. a) Define a group and prove that the identify element of group is unique. Again prove that the inverse of a group is unique.

   b) Let $s = \{0, 1, 2, 3, 4\}$. Show that S forms a group under the addition modulo 5.

Group 'B'

Attempt Any Five questions. **[5x10=50]**

4. Solve by separating the variables : $\sqrt{1 - x^2}\, dy + \sqrt{1 - y^2}\, dx = 0$

5. Solve the homogeneous differential equation : $\frac{dy}{dx} = \frac{x^2 + y^2}{2x^2}$

Cont.......

6. Solve the partial differential equations (Any one).

a) $z = ax + by + a^2 + b^2$ b) $xp - yq + x^2 - y^2 = 0$

7. Solve : $(mz - ny)p + (nx - lz)q = ly - mx$

8. Test the convergent of the series and find its sum if convergent:

$$3 + \frac{3}{-4} + \frac{3}{(-4)^2} + \cdots\cdots$$

9. Test whether the given series below is absolutely convergent of conditionally convergent $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

10. Find the interval and radius of convergence of the power series : $1 + 2x + 4x^2 + 8x^3 + \cdots$

Good Luck!

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Back Exam-2076, Falgun/Chaitra

Program: **Diploma in Civil/ Hyd/Arc/Elx/IT/ Computer Engineering**

Full Marks: 80

Year/Part: **II/I [New Course]**

Pass Marks: 32

Subject: **Engineering Mathematics III**

Time: 3 hrs

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt **All** Questions.

1. a) Define trigonometric and Fourier series. Determine the Fourier coefficient ao by Euler's Formula. [1+1+3]

   b) Find the fourier series of the function [5]

$$F(x) = \begin{cases} 0 & 0 < x \le \pi \\ 1 & \pi < x < 2\pi \end{cases}$$

2. a) Define a group. In a group, prove that [5]
   $(a * b)^{-1} = b^{-1} * a^{-1}$
   Also prove that inverse of each element of a group is unique.

   b) Given a set G={0,1,2,3,4}and a binary operation addition modulo $5(+_5)$ is defined on G. Prepare caley's table for it. Find the identity and inverse element of 3 and 4 [5]

3. a) By using definition of partial derivatives find Fx nad Fy for $F(x, y) = x^2y - xy^3$ [5]

   b) If $U = \sqrt{x^2 + y^2 + z^2}$ then prove that. [5]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$$

Group 'B'

Attempt **Any Ten** Questions.

4. Solve $(1 + cosy)dy = (1 - cosx)dx$ [5]

5. Solve $x\frac{dy}{dx} = y - x\tan {}^{y}/_{x}$ [5]

6. Form a P.D.E $z = \phi(x + iy) + \phi(x - iy)$ [5]

7. By using ratio test, test the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(2n-1)!}$ [5]

8. Test the following series for convergence by Cauchy root test. $x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \ldots\ldots\ldots\ldots$ [5]

9. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-3)^n}{10^n}$ [5]

10. Define periodic function and its period. Find the least period (fundamental period) of $Tan\ \frac{3x}{5}$ [5]

11. Find the Taylor's series expansion of $f(x) = \frac{1}{1-x}$ at $x = 0$ [5]

12. Find $\frac{du}{dt}$ of $U = e^{xyz}, x = t^3,\ y = \frac{1}{t}, z = e^t$ [5]

13. A binary operation is defined on the set of real numbers as below: $.m * n = 2mn - m + 3n$ for all $m, n\ ER$. [5]
Find the identity element and inverse element of 2 and - 4

Good Luck!

Council for Technical Education and Vocational Training
Office of the Controller of Examinations
Sanothimi, Bhaktapur
Regular/Back Exam-2075, Falgun/Chaitra

Program:	**Diploma in Civil/ Architecture/ Computer/ Electronics/ IT Engineering**	**Full Marks: 80**
Year/Part:	**II/I (New+Old Course)**	**Pass Marks: 32**
Subject:	**Engineering Mathematics-III**	**Time: 3 hrs**

Candidates are required to give their answers in their own words as far as practicable. The figures

Group 'A'

Attempt All questions. **[3x(5+5)=30]**

1. a) Define partial derivative of a function. Using definition, Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$: $f(x,y) = xy + y^2$

   b) Define homogeneous function. State and prove Euler's theorem for a two variable cases.

2. a) Define ordinary differential equation. Solve by separation of variables (any One)

   i) $\frac{dy}{dx} = \frac{x+y}{x+y+1}$ ii) $\frac{dy}{dx} + 1 = e^{x+y}$

   b) Show that the given function is homogeneous and solve:
   $x \sin\frac{y}{x} dy = \left(y \sin\frac{y}{x} - x\right) dx$

   OR

   Show that the given equation is exact and solve:
   (2ax+by) ydx+ (x+2by) xdy=0

3. a) Discuss the convergence of given geometric series for r=1, -1 and $|r| < 1, |r| > 1$. $a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$

   (b) Test the convergence of series by comparison test or ratio test (Any One):

   i) $\sum \frac{\sqrt{n}}{n^2+1}$ ii) $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots \ldots \ldots \ldots$

Contd..........

Group 'B' [10x5 =50]

Attempt All questions.

4. Find $\frac{dy}{dt}$ (Any One):

i) u= z+ sin (xy), x=t, y=log t, z=e^{t-1}

ii) u= $x^3 - y^3$, x= cost, y=sin t

5. Form a partial differential equation: $lx + my + nz = f\,(x^2 + y^2 + z^2)$

6. Solve the partial differentiate equation: $y^2p - xyq = x(z - 2y)$

7. Find the interval and radius of convergence of the power series:

$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$

8. Find the Taylor's series expansion of f(x)=$\sqrt{x}$ about x=0.

9. Obtain the Fourier series: $f(x) = \begin{cases} 0, -2 < x < 0 \\ 2, 0 < x < 2 \end{cases}$

10. Find the fourier series for the function defined as f(x) =$\begin{cases} \pi, -1 < x < 0 \\ -\pi, 0 \le x < 1 \end{cases}$

11. Define binary operation. Show that multiplication (x) is binary on the set s={1,w,w^2}, where w is the cube root of unity.

12. Find the identity element for the binary operation is defined as x*y=x+y=1 for every x, y∈ R. Also find the inverse of 2 and -3.

13. Let G=R- {-1}, the set of real numbers without -1. An operation* is defined on G by x*y = x+y+xy for all x, y ∈ G. Show that (G,*) is a group.

Good Luck

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Back Exam-2074, Falgun/Chaitra

Program:	**Diploma in Civil/Architecture/ Computer/Electronics Engineering & Information Technology**	**Full Marks: 80**
Year/Part:	**II/I (New + Old Course)**	**Pass Marks:32**
Subject:	**Engineering Mathematics III**	**Time: 3 hrs**

Candidates are required to give the... practicable. The figures in the marg...

Group "A"

Attempt (All) questions **[10x3=30]**

1. Define Fourier series in the interval $(-\pi, \pi)$. Find the Fourier series of the function.

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \le x < \pi/2 \\ 0 & \pi/2 \le x < \pi \end{cases}$$

2. a) Using Maclaurin's series, expand the function f(x)=tan x.

   b) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

3. a) If $(x, y) = x3 + 3x2y + 3xy2 + y3$, Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$.

   b) Find total differential $\frac{du}{dt}$ if : $u = (x + y)e^{xy}, x = t, y = \frac{1}{t^2}$.

Group "B"

Attempt (All) questions **[10x5=50]**

4. **Solve:** $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$

5. Solve the homogeneous differential equation: $\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$

6. Show that the equation: $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$. Is exact and solve it.

7. Discuss the convergence of the series:

$$x + \frac{3x^2}{5} + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \cdots \ldots\ldots + \frac{n^2-1}{n^2+1}x^n + \cdots \ldots\ldots$$

8. Show that the series: $1 - \frac{2}{3} + \frac{3}{3^2} - \frac{4}{3^3} + \ldots\ldots\ldots\ldots$ Is absolutely convergent.

9. Form a partial differential equation by eliminating f(x) from the equation $x + y + z = f(x^2 + y^2 + z^2)$.

10. Solve the partial differential equation: $\frac{y^2 z}{x}p + zxq = y^2$

Contd...........

11 Find the smallest period of sin 5x.

12. Prove that:

a) The identify element of a group (G, •) is unique.

b) In a group, the inverse of an element is unique.

13. Given an algebraic structure ((G, •) with G=R- {1}, the set of real numbers without the unit number and • stands for the binary operation defined by: $x \bullet y = x + y - xy \ for \ all \ x, y \in G$. Find the identity and inverse elements of 3 and -2.

OR

Write the standard form the equations of hyperboloid of one sheet and elliptic paraboloid. Find the equation of the tangent plane to the ellipsoid. $\frac{x^2}{27} + \frac{y^2}{12} + \frac{z^2}{3}$.

Good Luck!

Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/ Back Exam- 2073, Falgun

Program: Diploma in IT/ Computer/ Electronics Engineering **Full Marks: 80**

Year/Part: II/I (New+Old DEX) **Pass Marks: 32**

Subject: Engineering Mathematics-III **Time: 3 hrs**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt All questions.

Group 'A' **[3x(5+5)=30]**

1. (a) Write the standard form the equations of hyperboloid of one sheet and elliptic paraboloid. Find the equation of the tangent plane to the ellipsoid $\frac{x^2}{27}+\frac{y^2}{12}+\frac{z^2}{3}$

   (b) If $u= x^3+ y^3 + z^3 -3xyz$, find $\frac{du}{dx}$, $\frac{d^3u}{dxdydz}$

2. (a) Partial differential equation. Solve: $\frac{dy}{dx}+1 = e^{x+y}$

   (b) Solve: $\frac{d^2z}{dx^2} - \frac{d^2z}{dy^2} = e^{x+2y}$

3. (a) Define p-series. Test convergence of the series.

$$\sum_{n=1}^{\infty}\frac{x^2}{3^x}$$

   (b) Find the taylor's series generated by $f(x) = \frac{1}{x-1}$ at x=2

Group 'B' **[10x5 =50]**

Attempt All questions.

4. Find the half range cosine series expansion of the function.

$$f(x) = \begin{cases} 1, & 0 < x < \frac{1}{2} \\ 0, & \frac{1}{2} \leq x1 \end{cases}$$

5. Define trigonometric series. Find the fundamental at period of sin2x.

6. Expand f(x) in the Fourier series if $f(x)=x^2$ $(0< x <2\pi)$

7. Find the Fourier transform of the function: $f(x) = \begin{cases} 1, & a < x < b \\ 0, & otherwise. \end{cases}$

8. Solve : $xz\frac{dz}{dx} + yz\frac{dz}{dy} = xy$

9. Solve: $\frac{dy}{dx} + \frac{CoSx}{Sinx}y = x$

10. Solve the following partial differential equations:

$$x(y-z)\frac{dz}{dx} + y(z-x)\frac{dz}{dy} = z(x-y)$$

11. Show that the series $1-\frac{2}{3}+\frac{3}{3^2}-\frac{4}{3^3}+$ .... is absolutely convergent.

12. Find the interval of convergence or divergence of following series:
$1+2x+4x^2+8x^3+$ ------------

13. Define Fourier series. Fin the Fourier series of f(x) defined in the interval (-2, 2) as $f(x)= \begin{cases} 2, & -2 \le x \le 0 \\ x, & 0 < x < 2 \end{cases}$

Good Luck

ENGINEERING MATHEMATICS-III

Examination 2074 Regular/Back

Special scholarship

Full marks: 80
Pass marks: 32

Candidates are re ... s far as practicable.

Attempt any Two questions from Group A and Three questions from Group B

Group A

1. a) If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 7 & 2 \end{bmatrix}$. Prove that $B^T A^T = (AB)^T$ [5]

   b) Find the acute angle between the lines whose direction cosines are connected by the relations $l + m + n = 0$ and $l^2 + m^2 + n^2 = 0$ [5]

2. a) Solve using row equivalent matrix method or Cramer's rule: [5]

   $x + 2y - 3z = 0$
   $2x - y + 3z = 4$
   $3x + 4y + 7z = 14$

   b) Prove that: $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$ [5]

3. a) Prove that (use vector method): $\cos(A+B) = \cos A \cos B - \sin A \sin B$ [5]

   b) Define collinear vectors. Prove that the three points with the following vectors are collinear: $\vec{i} + 2\vec{j} + 3\vec{k}, -2\vec{i} + 3\vec{j} + 4\vec{k}, 7\vec{i} + \vec{k}$ [5]

Group B

4. Define complex number and find the cube roots of unity. [5]

5. State De-Moivre's theorem and hence use it to find the square roots of $\frac{1}{2} + i\frac{\sqrt{3}}{2}$. [5]

6. Find the local maxima and local minima and point of inflection if exists: [5]

   $f(x) = 2x^3 - 3x^2 - 12x + 4$

7. Calculate the Karl Pearson's coefficient of correlation: [5]

X	8	4	12	6	10
Y	11	13	8	9	7

8. A stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5 ft/sec., how fast is the area growing with radius is 8 ft. [5]

9. Find the area of the region enclosed by $x^2 = 4ay$ and $x = y$. [5]

10. The probability that a student passes a mathematics test is $\frac{3}{5}$ and he passes both mathematics and a chemistry test is $\frac{1}{5}$. The probability that he passes at least one test is $\frac{19}{20}$. What is the probability that he passes the chemistry test? [5]

11. Calculate A.M., G.M. and H.M. from the following data: [5]

Marks	0–10	10–20	20–30	30–40	40–50	50–60
Number of students	5	7	18	10	8	4

12. Find the area enclosed by the circle $x^2 + y^2 = 64$ [5]

13. Determine the maximum value of the objective function $F(x, y) = x + y$ subject to the constraints $2x + y \leq 20$, $2x + 3y \leq 24$, $x \geq 0$, $y \geq 0$ (Use graph paper)

ENGINEERING MATHEMATICS-III

Examination 2073 Regular/Back

Time: 3 hrs. | Full marks: 80 | Pass marks: 32

Candidates are required to give their answers in their own words as far as practicable.

Attempt any Two questions from Group A and Three questions from Group B

Group A

1. a) Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y) = xy + y^2$.

   b) Solve the following differential questions by separating the variables.

   i) $\tan x \, dy + \tan y \, dx = 0$

   ii) $\frac{dy}{dx} = e^{x-y} + x^2 \times e^{-y}$

2. a) Define p-series with example. Determine whether the following series is convergent or divergent by comparison test $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \ldots\ldots\ldots$

   b) Find the Maclaurin's expansion of the function: $f(x) = \log(1 + x)$

3. a) Define periodic function. Find the fundamental period of function $f(x) = \cos 3x$.

   b) If $G = \{\ldots\ldots\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\ldots\ldots\}$, then prove that $(G, +)$ is a group.

Group B

4. Verify Euler's theorem for homogenous function if $u = \frac{x^2 + z^2}{xy + yz}$

5. Find $\frac{du}{dt}$ of (any one)

   i) $u = x^2 + y^2, x = \cos t + \sin t, y = \cos t - \sin t$ at $t = 0$

   ii) $u = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$

Solve: $\dfrac{dy}{dx} = \dfrac{x + y}{x + y + 1}$

Prepare clayey table for the set {0, 1, 2, 3, 4} under the operation addition modulo 5. Identify the identity element and the inverse of each element.

Show that the given equation is exact and solve;

$$(x + y - 1)\, dx + (x - y - z)\, dy = 0$$

Form P.D.E. by eliminating the form $lx + my + nz = f(x^2 + y^2 + z^2)$

10. Define Fourier series check whether the function, $f(x) = \begin{cases} -2x \text{ for } -\pi < x < 0 \\ -2x \text{ for } 0 < x < \pi \end{cases}$ is odd or even and hence obtain the corresponding Fourier series.

11. Show that the following series is divergent.

12. Define alternating series with example. Test whether the following series is absolute convergent or conditionally convergent $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots\dots\dots$

13. Find the interval of convergence and radius of convergence of the given power series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots\dots$

14. Find the Fourier series expansion of $f(x) = x, \ -2 < x < 2$

15. Define group prove that the identity element of group is unique. Also, prove that the inverse of group is unique.

ENGINEERING MATHEMATICS-III

Examination 2072 Back

Full marks: 80
Pass marks: 32

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt any Two questions from Group A and Three questions from Group B

Group A

1. a) Prepare Cayley table for the set {0, 1, 2, 3, 4, 5}. Under the operation multiplication modulo 6. Identify the identity element and the inverse of each element if possible.

   b) Define group. Prove that the identity element of group is unique. Also, show that the inverse of group is unique.

2. a) If $u = \sqrt{x^2 + y^2 + z^2}$, then prove that: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$

   b) Let $u = \frac{x^4 + y^4}{x + y}$, prove that: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$

3. a) Define the p-series. Test the series for convergence by apply ratio of $\frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \ldots\ldots\ldots$

   b) Find the interval and radius of convergence of the power series;
   $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\ldots$

Group B

4. Solve: $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$

5. Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

6. Solve the partial differential equations: (Any one)

   i) $z = ax + by + a^2 + b^2$

   ii) $xp + yq = z$

7. Show that the given equation is exact and solve:

   $$(2\,ax + by)\,y\,dx + (ax + 2\,by)\,x\,dy = 0$$

8. Using definition, find $\frac{\partial f}{\partial y}$ form $f(x, y) = x^2y$

9. Find the Maclaurin's series expansion of $\cos x$.

10. Find the smallest positive period p of $\sin nx$.

11. Find the Fourier series of given function in the given interval,

$$f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2, & 0 < x < 2 \end{cases}$$

12. Test whether the function is even or odd. Also, find the corresponding Fourier series;

$$f(x) = \begin{cases} -2x \text{ for } & -\pi < x < 0 \\ -2x \text{ for } & 0 < x < \pi \end{cases}$$

13. Find $\frac{du}{dt}$: $u = x^2 + y^2$; $x = 2t + 1$; $y = t^2 + 2$

ENGINEERING MATHEMATICS-III

Examination 2071 Regular/Back

New Course

Time: 3 hrs.

Full marks: 80
Pass marks: 32

Candidates are required to give their answers in their own words as far as practicable.
Attempt any Two questions from Group A and Three questions from Group B

Group A

1. a) Prepare Cayley table for the set {0, 1, 2, 3} under the operation multiplication modulo 4. Identify the identity element and the inverse of each element if possible.

   b) Define group. Prove that the identity element of group is unique. Also show that the inverse of group is unique.

2. a) Solve: $(x+y+1)\frac{dy}{dx}=1.$

   b) Show that the given equation is exact and solve;
   $(x+y-1)dx+(x-y-2)dy=0.$

3. a) Define convergent and divergent series. Test whether series is convergent or divergent;
   $1+\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\ldots\ldots\ldots$

   b) Show that the series is conditionally convergent;
   $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots\ldots$

Group B

4. Form a partial differential equations: (Any one)
   i) $z=ke^{ax}\sin ay$
   ii) $lx+my+nz=f(x^2+y^2+z^2)$

5. Solve the partial differential equations. [Any one]

i) $\frac{\partial z}{\partial x}xz + yz\frac{\partial z}{\partial y} = xy$

ii) $xp - yq + x^2 - y^2 = 0$

6. Find the interval and radius of convergence of power series. [Any one]

i) $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n(n+1)}{2}x^{n-1}$

ii) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$

7. Assuming the convergence of Taylor's series, find the Maclaurin's series expansion of sin x.

8. Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ from $f(x, y) = x^2 - xy$.

9. Find $\frac{du}{dt}$ (Any one);

i) $u = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$

ii) $x^2 + y^2 + z^2, x = 2t + 1, y = t + 5, z = 7t.$

10. Let $u = \sin^{-1}\frac{x^2 + y^2}{x + y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

11. Find the Fourier series of given function on the given interval;

$$f(x) = \begin{cases} 0, 0 < x < \pi \\ 1, \pi < x < 2\pi \end{cases}$$

13. Test whether the function is even or odd. Also find the corresponding Fourier series;

$$f(x) = \begin{cases} -2x, -\pi < x < 0 \\ 2x, 0 < x < \pi \end{cases}$$

Council for Technical Education and Vocational Training
Office of the Controller of Examination
Sanothimi, Bhaktapur

Regular/Back Exam Chaitra, 2069

Program: Diploma in IT / Computer / Electronics Eng. (New)

Year/Part: II/I

Full Marks: 80

Subject: Engineering Mathematics III

Pass Marks: 32

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt *All* questions.

1. a. Define the partial differential equation. Form a P.D.E by eliminating a and b from $z = (x + a)(y + b)$ [1+4=5]

   b. Obtain the general solution of $2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = x + y + 1$ [5]

2. What is periodic function and define the Fourier series. Find the Fourier series of the [1+2+7=10]

$$f(x) = \begin{cases} -1, & if\ -\pi < x < 0 \\ 1 & if\quad 0 < x < \pi \end{cases}$$

3. a. Use Maclaurin expansion, prove that [5]

$$Sin\, x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots, \ldots, \ldots$$

   b. Test the convergence and divergence of [5]

$$1 + 3x + 5x^2 + 7x^3 + \cdots, \ldots, \ldots$$

Group 'B'

Attempt Any Ten Questions. [5x10=50]

4. What is space curve? Find the equation of the tangent line at a given point P (r) on the curve C.

5. Define the central conicoid. Show that the plane $3x + 12y - 6z = 17$ tangent to the conicoid $3x^2 - 6y^2 + 9z^2 = 17$

6. Find the condition that the plane $lx + my + nz = p$ may touch the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

7. Solve: $(1 + x)\, ydx + (1 + y)x\, dy = 0$

8. Solve by reducing to exact form $2xy\, dy - y^2 dx = 0$.

9. Solve; $\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$.

10. Define even and odd function. Check even or odd of the function (i) $f(x) = x\cos x + \sin x$
(ii) $f(x) = x \sin x$

11. Define convergent of a sequence. Test the convergence and divergence of the series.
$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots\ldots$

12. Using the comparison test show that the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots$ is convergent.

13. Test whether the series $\sum_{n=1}^{\infty} \frac{n!n!}{(2n)!}$ converges or diverges.

14. Find the smallest period of $f(x) = \cos nx$.

15. Find the interval, radius and centre of convergent of infinite series:
$\frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots\ldots$

The End